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ORIGINAL ARTICLE

Problem solving in the United States, 1970–2008: research and theory, practice and politics

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Abstract Problem solving was a major focus of mathematics education research in the US from the mid-1970s though the late 1980s. By the mid-1990s research under the banner of ''problem solving'' was seen less frequently as the field's attention turned to other areas. However, research in those areas did incorporate some ideas from the problem solving research, and that work continues to evolve in important ways. In curricular terms, the problem solving research of the 1970s and 1980s (see, e.g., Lester in *J Res Math Educ*, 25(6), 660–675, 1994, and Schoenfeld in *Handbook for research on mathematics teaching and learning*, MacMillan, New York, pp 334–370, 1992, for reviews) gave birth to the ''reform'' or ''standards-based'' curriculum movement. New curricula embodying ideas from the research were created in the 1990s and began to enter the marketplace. These curricula were controversial. Despite evidence that they tend to produce positive results, they may well fall victim to the ''math wars'' as the ''back to basics'' movement in the US is revitalized.

1 Introduction: national context

It is important to understand the national context within which educational research and development (R&D) in the US take place. In some nations educational research and development are tightly coupled. In others, educational R&D is funded and orchestrated by a ministry of education or its equivalent. This is not the situation in the US. To appreciate the US context, four main points must be understood.

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1.1 Absence of a national curriculum

There is no national curriculum in the US. There have always been de facto curricula available from textbook publishers. However, most of the nation's approximately 15,000 individual school districts have, until recently, formally been free to pursue locally established instructional goals. Those goals have varied widely.

1.2 The independence of the research and development communities

Research and curriculum development in the US have been largely decoupled. With the exception of a major curriculum initiative in the 1990s funded by the US National Science Foundation (NSF) and recent attempts by the US Department of Education (DoEd) to determine ''what works,'' federal research initiatives have tended to focus on supporting basic research, and until recently have not attempted to direct research toward curriculum development or to the evaluation of curricula. In thematic terms, the NSF supported research on cognitive processes and problem solving in the 1980s, sociocultural research and cross-disciplinary ''theories of learning'' initiatives in the 1990s, and contextually grounded research such as *design experiments* in recent years. Typically NSF-funded research is considered ''basic'' and is not aimed at specific curricular goals. Since the year 2000, curricular leverage from the Department of Education has come in two ways: (1) an emphasis on particular methods of educational research (specifically, an insistence on the use of randomized controlled trials as the so-called ''gold standard'' for comparing and evaluating curricula), and (2) the implementation of a federal law (the ''No Child Left Behind'' Act, or NCLB) that has functioned to impose severe limits on what was, at one time, significant curricular autonomy granted to the nation's 50 states.

1.3 Local autonomy rather than central direction

There is ''distributed autonomy'' for curriculum development in the US. Funding for research in mathematics education comes largely from two federal agencies, the NSF and DoEd; in comparison, funding from private foundations is comparatively minor. But, there is not one set of national curricular goals. In fact, the idea of their being such a unitary set would trample on the American constitutional and political traditions of ''states' rights,'' traditions that grant the 50 states significant autonomy. Educational goal setting and standard-setting is left to the states. With one major exception, described at length below, the prevailing assumption has been that commercial publishers will produce textbooks in line with state standards, and that ''market forces'' will result in continuous improvements to those books. Moreover, commercial publishers produce text *series* for the elementary and middle grades (kindergarten through grade eight). Hence, most school districts will buy textbook *programs* consistent with their states' standards.

1.4 Homogeneity nonetheless

Despite the autonomy granted to all the states, there is significant homogeneity in the choices of curricula available to school districts across the US.

Three major states—California, Texas, and New York—contain, in toto, about one fourth of the US population. Those states are ''textbook adoption'' states: each state publishes guidelines for curricula, and has commissions that decide which texts meet those guidelines. School districts in the textbook adoption states receive subsidies only for the purchase of books on those states' list of approved curricular materials. Because books are expensive, few districts will purchase materials that are not on the approved lists. Commercial publishers are, understandably, unwilling to sacrifice the textbook markets in any of those three big states. Thus, they produce textbook series that ''meet the standards'' of California, Texas, and New York. Those series are then marketed nation-wide. Hence, except for ''reform-oriented'' curriculum materials (see below) there is relatively little variation in the materials available to students. Moreover the ''high stakes testing'' mandated by the No Child Left Behind act has resulted in many states creating straightforward, skills-oriented assessments. A universal trait, not just in the US, is that teachers will ''teach to the test.'' Hence, there are political pressures toward a narrowing of the curriculum, with the direction toward an emphasis on skills rather than concepts and problem solving.

2 Research and theory

2.1 Research overview

A substantial amount of mathematics education research in the 1960s was statistical in nature. The literature was dominated by ''treatment A versus treatment B'' comparison studies of the effects of educational interventions, factor analyses, correlational studies, and so on. For example, the early issues of the *Journal for Research in Mathematics Education*, which first appeared in 1970, consist almost exclusively of statistical studies. Late in the 1960s, a small number of researchers (e.g., Kilpatrick, 1967; Lucas, 1972; Kantowski, 1977), motivated by Pólya's (1945, 1954, 1981) writings on problem solving, began identifying the heuristic practices used by students in the act of solving problems. Early studies focused on correlations between the uses of various problem-solving strategies and problem-solving success. Later studies began to characterize problem-solving processes and their impact on problem solving success more directly. Problem solving research flourished in the US through the 1980s, and tapered off by the mid-1990s—although there was much more to be done.

Lester (1994) provides a broad synopsis of problem-solving research from 1970 through 1994. The following table, from Lester, summarizes major phases of problem solving research through the mid-1990s. The dates in the table are approximate (Table 1).

2.1.1 Results, 1970–1989

The importance of the knowledge base had never been in question. Beyond that, the following had been achieved by 1989. The field had worked out, in theory, the level of detail required for students to learn to employ heuristic problem-solving strategies of the type described in Pólya (1945). There was clear evidence that general heuristic strategies could be decomposed into families of more specific strategies, and that with appropriate instruction, students could learn to employ those strategies. The influence and importance of metacognition, especially of monitoring and self-regulation, had been established—not just in mathematical problem solving but also in all

Table 1 An overview of problem solving research emphases and methodologies: 1970–1994

Dates	Problem-solving research emphases	Research methodologies used
1970–1982	Isolation of key determinants of problem difficulty; identification of characteristics of successful problem solvers; heuristics training	Statistical regression analysis; early ''teaching experiments''
1978–1985	Comparison of successful and unsuccessful problem solvers (experts vs. novices); strategy training	Case studies; ''think aloud'' protocol analysis
1982–1990	Metacognition; relation of affects/ beliefs to problem solving; metacognition training	Case studies; ''think aloud'' protocol analysis
1990–1994	Social influences; problem solving in context (situated problem solving)	Ethnographic methods

Reprinted with permission from Lester (1994, p. 664)

non-routine intellectual performance. The role of belief systems in shaping people's problem solving behaviors (similarly, in all fields) had been documented. Moreover, the role of people's experiences with mathematics, both in and outside the classroom, was seen to be a primary shaper of people's beliefs and practices when they engaged in problem solving. And, there were existence proofs of successful problem-solving instruction.

That body of research—for details and summary, see Lester (1994) and Schoenfeld (1985, 1992)—was robust and has stood the test of time. It represented significant progress on issues of problem solving, but it also left some very important issues unresolved. On the ''applied'' side, it would have been a straightforward experimental matter to determine how much practice, of what kinds, would enable students to learn to use a wide range of problem-solving strategies. The theory had been worked out; all that needed to be done was the (hard and unglamorous) work of following through in practical terms. On the theoretical side, there was still a fundamental unresolved issue. The research to date said what was important to look at when people were engaged in problem solving and it provided explanations for success and failure, but it did not explain how and why people made the choices they did. That is, it offered a framework for characterizing problem solving, but it did not yet offer a theory of problem solving. To do so was an excruciatingly difficult task, simply beyond the capacity of the field to grapple with in the late 1980s. (There were broad psychological theories of knowledge acquisition and use, but there was nothing at the level of detail that could explain how and why people chose the particular paths they chose while engaged in problem solving.)

As Lester (1994) describes it, work in problem solving per se had dropped off significantly by the early 1990s. That the work is difficult and unglamorous is part of the reason it was not undertaken. Another part, Burkhardt and Schoenfeld (2003) argue, is the academic value system: working out a pragmatic program is given little credit, especially if the theoretical advances have been made by others. In addition, Lester (1994) argues that fads and fashions come and go in research; ''problem solving'' had been worked through, and in the 1990s ''sociocultural'' work had the same faddish appeal that problem solving had had a decade earlier.

2.1.2 Results, 1990-present

The drop-off in problem-solving research in the 1980s did not mean that there was no progress—only that it continued in a different guise. As will be seen below, the research that flowered through the 1980s did have a profound practical impact, in the creation of the National Council of Teachers of Mathematics' (1989) *Curriculum and Evaluation Standards for School Mathematics*, generally referred to as the *Standards.* The *Standards* emphasized problem solving, reasoning, making mathematical connections, and communicating with mathematics as important goals of instruction. Researchers began to design instruction aimed at these broader goals, often using methods called ''design experiments'' in which they created and explored the properties of instruction and the theories that lay behind it (see, e.g., Cobb, Confrey, diSessa, Lehrer & Schauble, 2003). Typically, design experiments were aimed at conceptual understanding—teaching some topic in a new way, either by means of a new approach to the content or the pedagogy, or both—rather than at problem solving directly. However, the close examination of productive learning environments has led to methodological and conceptual advances.

Recall that much of the original problem-solving research had been done in laboratory studies. The study of learning in classroom environments—especially in reform or standards-based classrooms which, although not necessarily focused on problem solving per se, were focused on mathematics as a sense-making activity—called for developing new analytical techniques and perspectives. Over the 1990s and into the twenty-first century researchers evolved a series of tools, techniques, and ideas for the characterization of productive learning environments; researchers began to develop tools and techniques for characterizing the mechanisms by which individuals developed in interaction with their environments, both in and out of the classroom. For a broad review of progress over the past 15 years, see Schoenfeld (2006a). There is not the space here for a broad review; in what follows I highlight some of the major themes related to problem solving.

2.1.3 Mathematics as sense-making

Magdalene Lampert's (2001) book, *Teaching Problems and the Problem of Teaching*, offers a broad view of her goals for teaching a year-long fifth-grade class, ranging from the content-related particulars of one segment of one lesson to her long-term goals (conceptual understanding, problem-solving competence, developing autonomy as learners, and personal growth) for her students, and the ways she sets about achieving those goals. This is *sense-making* writ large, a broad generalization of the themes that originated in the earlier research on problem solving. Research on standards-based curricula (see, e.g., ARC Center, 2003; Boaler, 2002; Senk & Thompson, 2003) provided clear evidence of the impact of standards-based instruction. Across the boards, comparisons of ''traditional'' (mostly skills-oriented) and standards-based instruction yielded the same results: students who studied from ''reform'' curricula performed about the same as students from traditional curricula on tests of skills, but performed significantly better than those students on tests of conceptual understanding, applications, and problem solving. (Some detail is given in the section of this paper entitled ''Data, during the math wars and to the present.'')

2.1.4 Discourse communities

At the process level, research explored the mechanisms by which productive classroom communities, typically embodying the values of reform, actually functioned. Such studies involve the examination of classroom discourse patterns (Cobb & Yackel, 1996; Horn, 2007; Lampert, Rittenhouse, & Crumbaugh, 1996; O'Connor, 1998; O'Connor & Michaels, 1993, 1996) and the ways on which those forms of interaction either support or inhibit the development of sense-making propensities in students. Important theoretical ideas that provide the conceptual superstructure for the examination of such classroom practices include the concept of *sociomathematical norms* and *accountability structures*. The idea of sociomathematical norms was introduced by Paul Cobb and Erna Yackel (Cobb & Yackel, 1996; Yackel & Cobb, 1996) to characterize patterns of ''taken-as-shared mathematical behavior'' in mathematics classrooms—for example, what constitutes an adequate explanation of a mathematical claim. (This is related to Guy Brousseau's notion of the *didactical contract*; see, e.g., Brousseau, 1997.)

2.1.5 Accountability structures

The idea of accountability structures, also related to the didactical contract, concerns the forms of responsibility involved in the classroom community. There is accountability to the mathematics—do students produce arguments that are rigorous and that are consistent with the ways in which mathematics is conducted? (For example, do they make and justify conjectures on solid mathematical grounds?) There is accountability to other students—to take them and their ideas seriously. And, there is accountability to the teacher—both in terms of the traditional authority structure, but also in that the teacher is the prime orchestrator of the classroom mathematical community, and a representative of the mathematical community in the classroom. These ideas have been explored by Ball and Bass (2003), Boaler (2007), and Horn (2007).

2.1.6 Productive classroom cultures

Some of themes described in the preceding paragraphs, all having to do with sense-making communities in classrooms, have been abstracted by Engle and Conant (2002). In a review that covers what might be called ''sense-making instruction in science and mathematics classrooms,'' Engle and Conant argue that there are substantial consistencies in the highly productive learning environments they examined. Common characteristics of those environments are:

- *Problematizing*: students are encouraged to take on intellectual problems
- *Authority*: Students are given authority in addressing such problems
- *Accountability*: Students' intellectual work is made accountable to others and to disciplinary norms
- *Resources*: Students are provided with sufficient resources to do all of the above. (Engle & Conant, 2002, pp. 400–401).

Such environments are rare, in part because of the pedagogical challenges of implementing them. Nonetheless, they do represent significant progress—''existence proofs'' are important.

But what of the two unresolved issues discussed above? The first, ''applied'' issue—how much practice, of what kinds, would enable students to learn to use a wide range of problem-solving strategies?—has simply not been pursued. As noted earlier in this article, such applied work is neither glamorous nor likely to contribute to faculty's advancement at research-oriented universities. In my opinion, this is a great shame. Crafting instruction that would make a wide range of problem-solving strategies accessible to students would be a very valuable contribution, and it would add significantly to the instructional progress achieved by the standards-based curricula. This is an ''engineering'' task rather than a conceptual one. The methods for decomposing complex heuristic strategies into families of simpler, learnable strategies are known; it is ''merely'' a matter of effort to follow through. It is most unfortunate that the incentive systems do not exist to attract people to this enterprise. (There is also the matter of politics: see the section on politics and curriculum adoption.) The story is different with regard to theoretical issues, as described immediately below.

2.2 Theory

I now take stock on the theoretical side, building on the summary in Lester (1994). As noted above, there had been a significant drop-off in studies on problem solving after the 1980s. On the one hand, that was not necessarily a bad sign. As the preceding discussion indicates, the field has continued to evolve. Research has continued to be concerned with sense making in mathematics. Research studies have moved from the laboratory to the classroom, and the field has developed some tools and techniques for grappling with the creation and the analysis of environments intended to foster sense making. (Recall that the four ''process standards'' of NCTM's 1989 *Standards* involved problem solving, reasoning, making mathematical connections, and communicating with mathematics.)

2.2.1 Progress

A major theoretical challenge has been to move from structural descriptions—''what affects success or failure in problem solving?''—to theoretical descriptions and explanations of how and why people make the choices they do while engaged in (mathematical) problem solving. To reframe the problem slightly, mathematical problem solving can be seen as a goal-directed activity. The main goal, if accepted by the individual, is to solve the problem. Typically many subgoals or alternative goals get generated, often as a function of what the individual believes will be productive, and knowledge is accessed and brought to bear in attempts to meet those subgoals. As subgoals are met (or not), other goals replace them and other knowledge is accessed, until either the problem is solved or the individual gives up. In similar ways, mathematics teaching can be seen as a goal-oriented problem-solving activity. A teacher begins a day, a week, or a year with a set of goals to achieve; over the course of time the teacher accesses relevant knowledge and establishes new goals in line with his or her values and beliefs. Thus, a theory of teaching is in essence a theory of problem solving.

The Teacher Model Group (TMG) at the University of California at Berkeley has, over the past two decades, developed a theory of teaching that has as its fundamental objective a theoretical answer to the question of how and why teachers make the choices they do while engaged in teaching (see, e.g., Schoenfeld, 1998; 2002). The core idea in this work is that a teacher's actions can be modeled as a function of his or her knowledge, goals, and belief and value systems. A number of different teachers, with very different styles, have been modeled. The theory appears to be robust, and it appears likely that it will apply, without significant modification, to many if not all goal-directed behaviors, specifically to the characterization of mathematical problem solving (see Schoenfeld, 2006b). This, in itself, will not yield a complete theory of problem solving, but it will represent a new plateau. The next set of questions to be confronted will be related to the need to integrate the sociocultural and the cognitive. How does context play into the choice of goals, the establishment of values and beliefs? How is identity shaped over time as a function of experience, and membership in various communities of practice (see, e.g., Wenger, 1998)? How are knowledge, beliefs, and values shaped over time? How does identity shape goal formation? And, how does all of this square with emerging research from neuroscience regarding the development and organization of knowledge? These will be very difficult questions, but—despite the fact that such issues have been ''off the radar screen'' for much of mathematics education over the past 20 years—there is reason to be optimistic both in terms of the progress that has been made and because progress continues to be made on broad issues of context and learning.

3 Curricula (and the politics that affect curriculum adoption)

3.1 A history of philosophical conflict

As noted in the introduction, school systems in the US are decentralized. Each of the 50 states has its own independent state board of education, which sets its own rules and regulations; these regulations (until recently, at least) gave local school districts a significant amount of latitude. In addition, there have historically been very different social/philosophical perspectives regarding the role of schooling (and thus mathematics) in American society (Rosen, 2000; Stanic, 1987). Among the goals and purposes espoused for mathematics instruction—some of which are clearly contradictory—are:

- *mathematics education for democratic equality*, the idea that schools should prepare students, both in terms of knowledge and in terms of social values, for participation in America's democratic society;
- *mathematics education for social efficiency*, the idea that schools should serve primarily as the training grounds for America's work force, preparing the majority of workers for low-end jobs;
- *mathematics education for social mobility*, the idea that schools should ''level the playing field'' by providing students with the kinds of skills that would enable them to climb social and economic ladders in the American meritocracy;
- *mathematics education for national defense*, the idea that schools should identify and nurture those students who are mathematically and scientifically talented, in order to insure the nation's economic, technological, and military supremacy;
- *mathematics education as an introduction to powerful ideas*, the idea that schools should provide an introduction to the power and beauty of mathematics.

3.2 Pendulum swings

The curriculum-related story of problem solving in the US over the past half century is one of pendulum swings, as the pragmatic foci of mathematics instruction moved back and forth between teaching for understanding (TFU) on the one hand and teaching for mastery (TFM) on the other. Through the 1950s, the ''traditional'' curriculum held sway. Arithmetic was studied in grades K-8, ''algebra 1'' in grade 9, Euclidean geometry in grade 10, ''algebra 2'' (and sometimes trigonometry) in grade 11, and ''math analysis'' (preparation for calculus) in grade 12. That is, students took those courses if they continued mathematics courses. Through the 1980s and into the 1990s, most states required that students take only 1 or 2 years of mathematics in order to qualify for a high school diploma. From grade 9 on, when mathematics became optional, half of the students left mathematics each year. The content of the traditional curriculum was mostly procedural, although it did have a conceptual underpinning. Students were taught various procedures (e.g., solving linear or quadratic equations; graphing pairs of simultaneous equations or solving them analytically; performing geometric constructions or proving geometric theorems) and then given large numbers of exercises on which to master the relevant skills.

The American mathematics curriculum has—until recently—received little public attention except in times of perceived national crisis. Thus, for example, ''In the 1940s it became something of a public scandal that army recruits knew so little math that the army itself had to provide training in the arithmetic needed for basic bookkeeping and gunnery. Admiral Nimitz complained of mathematical deficiencies of would-be officer candidates and navy volunteers. The basic skills of these military personnel should have been learned in the public schools but were not'' (Klein, 2003). As noted, the ''fix'' in this case was in the military, not in the schools: Nimitz's complaint did not give rise to curricular reform.

The next crisis did affect schooling in serious ways. Amidst the ''cold war,'' Russia's 1957 launching of the satellite Sputnik energized the American scientific establishment, which saw itself as falling behind the Soviet Union scientifically and, in ways it could not afford, militarily. The mathematical and scientific communities joined in the effort to update mathematical and scientific curricula. The result in mathematics was the ''new math.'' The popular perception is that the new math was a curricular disaster[1]. The nation's elementary school teachers, who tend not to be mathematically sophisticated, were asked to teach bodies of mathematics (e.g., aspects of set theory, logic, and modular arithmetic) that they had not studied and with which they were not comfortable. When homework was assigned, the nation's parents found themselves confronting unfamiliar topics; they had great difficulty helping their children to solve problems that they found alien and unmotivated. The net result was a backlash, the ''back to basics'' movement, which swept most of the new math out of America's classrooms. A rote, ''basics'' approach dominated American classrooms through much of the 1970s.

3.3 Problem solving as a vacuous curriculum focus in the 1980s

In 1980 the National Council of Teachers of Mathematics, the US's professional organization for mathematics teachers, issued a small pamphlet entitled *An Agenda for Action: Recommendations for School Mathematics of the 1980s.* That document summarized the 1960s and 1970s as follows:

[1] As is often the case, such statements are over-simplifications. Many of the ideas from the post-Sputnik curricula (e.g., ''hands-on science'' and aspects of problem solving) took hold over succeeding decades, and a new generation of researchers in mathematics education came into being because of the reaction to Sputnik.

> In the 1960s there was considerable ferment in mathematics curriculum and instruction. Although public attention was focused on the more visible attempts at program revision, we are aware two decades later that change was more apparent than real.
>
> In the 1970s the concern of the public was directed toward problems evidenced almost exclusively in tests scores. Schools have responded to this concern in a variety of ways, but a clear-cut and carefully reasoned sense of direction that looks toward the future has been lacking. (NCTM, 1980, p. i)

The direction proposed by NCTM was problem solving. The *Agenda for Action* made eight recommendations, the first of which was: ''Problem solving must be the focus of school mathematics in the 1980s.'' It went on to elaborate this statement with a series of recommended actions:

- The mathematics curriculum should be organized around problem solving….
- The definition and language of problem solving in mathematics should be developed and expanded to include a broad range of strategies, processes, and modes of presentation that encompass the full potential of mathematical applications….
- Mathematic teachers should create classroom environments in which problem solving can flourish…
- Appropriate curricular materials to teach problem solving should be developed for all grade levels….
- Mathematics programs … should involve students in problem solving by presenting applications at all grade levels…
- Researchers and funding agencies should give priority … to investigations into the nature of problem solving and to effective ways to develop problem solvers. (NCTM, 1980, pp. 1–5)

I quote these recommendations at length because they are every bit as relevant and appropriate today as they were in 1980, when they were produced. Here is what happened.

For the most part, the first five of the recommendations above were ignored over the course of the 1980s. ''Problem solving'' did become a fashionable term, but its implementation in most classrooms was a travesty. Commercial publishers adopted ''problem solving'' editions of their textbooks, but for the most part those editions were trivial modifications of their earlier, drill-oriented texts. The main changes in the new problem solving editions were rhetorical. Although the new texts typically invoked Pólya's name and described the four ''stages'' of problem solving (understanding the problem, making a plan, executing the plan, and ''looking back'') from Pólya's (1945) *How to Solve It*, the actual contents of the texts remained largely the same. In practice, ''problem solving'' usually meant solving routine one- or two-step word problems such as

> John had eight toy trucks. He gets four more toy trucks. How many toy trucks does John have all together?
>
> or
>
> John had $5.00. He bought a pen for $.39 and a notebook for $2.19. How much money does John have left?

In sum, ''problem solving'' in American classrooms in the 1980s came to mean ''solving (simple) word problems.'' This was for three main reasons. First, as the research chronology indicates, problem-solving research was still in its infancy when NCTM issued *An Agenda for Action.* Hence there was little research to guide curriculum development (or even curriculum goals) at that time. Second, teachers are a conservative force: significant change almost always encounters significant resistance, and it takes time and effort to implement change, even when that change is considered desirable. Third, the mechanics of the publishing industry militate against meaningful change. As noted in the introduction, publishers produce mathematics textbook series for grades K-8. These series are usually produced by teams of authors. The lead authors produce specifications for what the content of various sections will be, and large numbers of authors often fill out the rest of the content. This kind of ''production line'' approach to textbook manufacture is efficient: a publisher can produce a new series in a relatively short amount of time, in the same way that an automobile manufacturer can produce a new model each year. However, the process is also constraining: just as in the automobile industry, the current product is largely grounded in the previous product. Significant model changes are very expensive and they come rarely. In the textbook industry, there was a rush to get out ''problem solving'' editions in the immediate aftermath of the *Agenda.* The changes in curricula were thus ''evolutionary'' rather than revolutionary; they were superficial rather than substantial.

3.4 Political context in the 1980s

Over the course of the 1980s, the research base regarding problem solving became much more robust. And, the nation's competitiveness became an issue once again—this time in economic rather than military terms. Through the 1970s and 1980s the American economy faltered, while Japan's economy grew strong. A very influential report, *A Nation at Risk: The Imperative for Educational Reform* (National Commission on Excellence in Education, 1983) portrayed the challenges in stark terms:

> Our Nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world.… The educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people….
>
> If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. As it stands, we have allowed this to happen to ourselves. We have even squandered the gains in student achievement made in the wake of the Sputnik challenge. Moreover, we have dismantled essential support systems, which helped make those gains possible. We have, in effect, been committing an act of unthinking, unilateral educational disarmament. (National Commission on Excellence in Education, 1983, p. 1)

In this context, there was a greater likelihood of response to calls for curricular change. The US National Research Council formed the Mathematical Sciences Education Board (MSEB), which was intended to provide continual attention to issues of mathematics education, rather than the periodic crisis-driven attention that mathematics education had hitherto received. In early 1989 MSEB produced a report, *Everybody Counts*, that called for serious national attention to the reform of mathematics education in the US. This was reform along multiple dimensions, including: the goals of mathematics instruction (quantitative literacy as well as the production of the mathematical and scientific elite), demographics (historically, failure and drop-out rates from mathematics for Latinos, African Americans, and Native Americans had been very high; *Everybody Counts* called for addressing this loss of human potential) and mathematics content (the ''traditional'' curriculum had been problematic; it was time to open things up.)

3.5 New goals for students

Soon after the publication of *Everybody Counts*, the National Council of Teachers of Mathematics produced the (1989) *Curriculum and Evaluation Standards for School Mathematics.* The *Standards*, as they are known, are not a research document. They were, however, produced by people who knew the research on problem solving. The problem-solving research was incorporated into the goals for instruction, called ''standards.'' The first four NCTM standards—desired outcomes of mathematics instruction—concerned the mathematical processes that had been at the core of mathematical problem solving research over the preceding decades: mathematics as problem solving; mathematics as communication; mathematics as reasoning; and mathematical connections. The *Standards* promoted new societal goals: ''(1) mathematically literate workers, (2) lifelong learning, (3) opportunity for all, and (4) an informed electorate. Implicit in these goals is a school system organized to serve as an important resource for all citizens throughout their lives.'' (NCTM, 1989, p. 3) They explicitly promoted new goals for students.

> The K-12 standards articulate five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically. These goals imply that students should be exposed to numerous and varied interrelated experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess, and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write, and discuss mathematics; and that they should conjecture, test, and build arguments about a conjecture's validity. (NCTM, 1989, p. 5)

NCTM has tried to remain true to its vision. In 2000 NCTM published a successor to the 1989 *Standards*, called *Principles and Standards for School Mathematics* (NCTM, 2000). *Principles and Standards*, as it is known, was an attempt to take stock—to reflect on lessons learned in the decade since the original *Standards* were published, to incorporate research progress over the previous decade, and to update in terms of changes in context, such as the advancement of technology. In 2006, NCTM produced *Curriculum Focal Points for Kindergarten through Grade 8 Mathematics* (NCTM, 2006). (See the concluding pages of this article for a discussion of that document. For now, I continue with the chronological narrative.)

The 1989 *Standards*—with the help of the US National Science Foundation (NSF)—changed the political and curricular landscape. As noted above, publishers typically produce textbook series. Producing and marketing such series is expensive: during the adoption process for the 1992 California Mathematics Framework, representatives from major publishers stated that the development and marketing of a new curriculum series costs on the order of $25 million, far too much for the publishers to spend on experimental ideas. Simply put, curricula embodying the ideas of the *Standards* would not be developed by commercial publishers.

3.6 New curricula

Recognizing that the commercial marketplace was not likely to produce *Standards*-based curricula, the NSF, from 1989 through 1991, issued a series of requests for proposals (RFPs) for the development of mathematics curricula aligned with the *Standards.* NSF provided the funding for the development of very different mathematics curricula at each of three grade spans: elementary (typically some continuous subset of grades from kindergarten through grade 6), middle (typically grade 5 or 6 through grade 8 or 9) and secondary (typically grades 9 through 12).

With the help of NSF funding, reform or standards-based curriculum developers were able to produce curricular materials without needing to depend on corporate support. Once the materials were produced, their developers were free to market them through commercial publishers. In this way, new curricular materials could make their way past the publishers' cost threshold. Of course, this whole process took time. The RFPs were issued from 1989 through 1991, after which the proposal writing and evaluation process needed to run their course. Typically, curriculum developers needed n years of development time to develop n years of curriculum—and these, of course, were the alpha versions. Those were completed in the mid-1990s. Beta versions soon followed. The first groups of students to have experienced an entire elementary, middle school, or secondary standards-based curriculum emerged from those curricula about the year 2000. Hard data on the impact of these curricula began to appear about the same time.

3.7 The math wars

So, what happened in the interim? In brief: the *math wars*, the most vicious public battle over mathematics curricula in US history. The wars started in California, which was at the cutting edge of reform. As noted above, California, with approximately 10% of the US population, is a textbook adoption state. Roughly every 7 years a committee is appointed to write a Mathematics Framework, which describes curricular goals for California students. From this Framework, a set of ''textbook adoption criteria'' is abstracted. Only those textbooks that are declared, after review, to meet these criteria are then eligible for state subsidies when school districts buy their textbooks. In short, millions of dollars are at risk for publishers during each new adoption cycle. Books on the state adoption list will reap huge profits; books not on the adoption list will ''starve,'' because school districts do not, in general have the surplus funds to purchase books that are not on the adoption list.

The 1985 *California Mathematics Framework* (California State Department of Education, 1985) was considered a progressive document—an antecedent of the 1989 NCTM *Standards.* California's professional teacher organization, the California Mathematics Council, was one of the most progressive teacher organizations in the country, and one of the most enthusiastic adopters of the spirit of the 1989 *Standards.* When the next adoption cycle came, the 1992 *California Mathematics Framework* (California State Department of Education, 1992) ''pushed the envelope'' a good deal further: it emphasized reform, focusing on ''mathematical power'' and collaborative and independent student work while de-emphasizing traditional skills and algorithms.

Aspects of the 1989 *Standards* had already raised hackles among conservatives. The *Standards* were not a complete curriculum outline. Rather, the *Standards* outlined a broad philosophical approach to curriculum, leaving a great deal to the imagination of potential curriculum developers. (Rather bluntly, Apple (1992) referred to the *Standards* as a ''slogan system.'') One of the most controversial aspects of the *Standards* was a series of lists of topics that should receive ''increased attention'' or ''decreased attention.'' Among the topics to receive increased attention in grades K-4 were ''use of manipulative materials'' and ''cooperative work''; among the topics to receive decreased attention were ''rote practice, rote memorization of rules, one answer and one method, use of worksheets, written practice, teaching by telling'' (pp. 20–21). In grades 5–8, ''actively involving students individually and in groups exploring, conjecturing, analyzing, and applying mathematics'' were among the topics to be emphasized; memorizing and manipulating formulas, teaching computations out of context, drilling on pencil-and-paper computations, stressing memorization, and [teacher] ''being the dispenser of knowledge'' were to be de-emphasized (pp. 72–73). In grades 9–12, computer utilities and scientific calculators were to be emphasized (among other things) while the traditional approaches to algebra word problems, operations on algebraic expressions, two-column geometric proofs, and trigonometric identities would receive less emphasis (pp. 126–127). Moreover, teachers were instructed to pay more attention to the ''active involvement of students in constructing and applying mathematical ideas,'' ''effective questioning techniques that promote student interaction'' and ''the use of a variety of instructional formats (small groups, individual explorations, peer instruction, whole-class discussions, project work)'' while paying decreased attention to ''teacher and text as exclusive sources of knowledge, rote memorization of facts and procedures, instruction by teacher exposition…'' (p. 129).

There are reasonable interpretations of these statements, and there are unreasonable ones. Curriculum developers and teachers made some of each. Textbooks containing some rather dubious mathematics entered the marketplace, and the *Standards* were blamed for them; the *Standards* were caricatured as abandoning mathematical values such as proof and replacing them with ''discovery'' and exploration''; and the *Standards* were accused of abandoning skills development to have students become ''mindless button-pushers'' when they used calculators and computers to get answer to problems. Moreover, some teachers' organizations made things worse by claiming a teacher should be a ''guide on the side'' rather than a ''sage on the stage.'' This led to claims by those opposed to reform that teachers were being urged to abandon their responsibilities as teachers!

It is difficult to convey the vehemence and the viciousness of the anti-*Standards* movement, which was born in California. Details can be found in Becker & Jacob (2000), Jackson (1997a, b), Jacob (1999, 2001), Jacob & Akers (2003), and Rosen (2000); an overall summary and possible explanations for some of the viciousness may be found in Schoenfeld (2004). However, readers should examine some of the anti-reform web sites directly (examples are ''mathematically correct,'' (http://www.mathematicallycorrect.com/); ''Hold Open Logical Debate (HOLD),'' (http://www.dehnbase.org/hold/); New York City HOLD, (http://www.nychold.com/)) to see just how nasty things have been. Anti-reformers referred to standards-based mathematics as ''fuzzy math'' and ''fuzzy crap.'' Invoking the failures of the new math, they christened reform the ''new-new-math'' and began a campaign to discredit it. Newspaper columnists hostile to reform wrote columns titled ''New-New Math: Boot Licking 101'' (Saunders, March 13, 1995) and ''Creatures From The New-New Math Lagoon'' (Saunders, September 20, 1995). Skilled in public relations, those opposed to the *Standards* created websites that galvanized the anti-reform movement, fomented parental opposition to reform curricula, and enlisted conservative politicians in drive to bring the state and the nation ''back to basics'' once again. With the help of right-wing politicians, traditionalists arranged for advocates of direct, skills-based instruction to be the *only* researchers allowed to testify before the California State Board of Education regarding research findings. Not surprisingly, the State Board was told that ''basics'' count and that the only proven form of instruction is direct instruction. Under pressure from the right wing, which was strongly represented on the State Board of Education, the Department of Education had begun a revision to the *California Mathematics Framework* earlier than typically scheduled; but the State Board, unhappy with the reformist tenor of the new draft, took over the process itself. The result, written in 2 weeks in 1997, was a return to ''basics'' that drove California policy for the next decade (and still does). California State Superintendent of Education Delaine Easton characterized it as follows:

> [The board's proposal] represents ... a decided shift toward less thinking and more rote memorization… They've deleted verbs like model, understand, estimate, interpret, classify, explain and create and the verb they most commonly substituted was compute… Essentially, this comes down to that we are going to teach kids to add, subtract, multiply and divide and we're not even going to let them use a calculator before the sixth grade. (reported in the Los Angeles Times, December 1, 1997)

The anti-reform movement spread from California to become nation-wide, with widespread attacks on standards-based materials. The tone of the attacks was uncompromisingly vicious. In January 1998, Richard Riley, the Secretary of Education of the US, gave a keynote talk at the Joint Mathematics Meetings (the annual meeting of the American Mathematical Society, Mathematical Association of America, Association for Symbolic Logic, Association for Women in Mathematics, National Association of Mathematicians, and Society for Industrial and Applied Mathematics). He was introduced by American Mathematical Society President Arthur Jaffe, who called the Secretary's appearance at the Mathematics Meetings a ''historic event.'' The main thrust of Riley's talk (a transcript of which is given in Riley, 1998) was that public conflict over mathematics curricula had gotten out of hand:

> I will talk in more detail shortly about these so-called ''math wars'' in California and elsewhere. But let me say right now that this is a very disturbing trend, and it is very wrong for anyone addressing education to be attacking another in ways that are neither constructive nor productive.
>
> It is perfectly appropriate to disagree on teaching methodologies and curriculum content. But what we need is a civil and constructive discourse. I am hopeful that we can have a ''cease-fire'' in this war and instead harness the energies employed on these battles for a crusade for excellence in mathematics for every American student (Riley, 1998, p. 488).

However, Riley's call for civility went unheeded, and hostilities increased. Indeed, Riley soon found himself in the middle of the math wars. The US Department of Education produced a report entitled *Exemplary and Promising Mathematics Programs* in 1999. The document, reporting the results of an expert panel convened to judge mathematics curricula, identified five ''exemplary'' and five ''promising'' mathematics curricula. All of the curricula were standards-based, and all hell broke loose when the report was published. A collection of roughly 200 mathematicians and scientists, including some Nobel Prize winners, signed an open letter, published in the Washington Post, decrying the awards and urging the Secretary to rescind them. It is worth noting, as Ralston (2004) did, that the odds that even a few of those who signed the open letter had more than a passing familiarity with even two or three of the ten curricula, much less all ten, were essentially zero. But this was no intellectual debate: this was war!

3.8 Data, during the math wars and to the present

It is important to recognize that the anti-reform campaign was waged on the basis of anecdote and innuendo, and in the absence of any firm evidence. As I noted above, the alpha versions of standards-based curricula were completed in the mid-1990s; the first cohorts of students to emerge from complete versions of the standards-based curricula did so around the turn of the twenty-first century. Thus, when the open letter urging Secretary Riley to rescind the ''exemplary'' and ''promising'' designations of the ten standards-based curricula was published, there was no evidentiary basis for the requests.

Since that time, the data have come in. The most comprehensive overview of standards-based curricula is Senk and Thompson (2003), which contains evaluations of all of the NSF-supported standards-based curricula. Putnam (2003) summarized the evaluations of four elementary curriculum evaluations as follows:

> [These four curricula] … all focus in various ways on helping students develop conceptually powerful and useful knowledge of mathematics while avoiding the learning of computational procedures as rote symbolic manipulations… Students in these new curricula generally perform as well as other students on traditional measures of mathematical achievement, including computational skill, and generally do better on formal and informal assessments of conceptual understanding and ability to use mathematics to solve problems. These chapters demonstrate that ''reform-based'' mathematics curricula can work (Putnam, 2003, p. 161).

Similarly, Chappell (2003) distills the results regarding standards-based middle-school curricula:

Collectively, the evaluation results provide converging evidence that Standards-based curricula may positively affect middle-school students' mathematical achievement, both in conceptual and procedural understanding…. They reveal that the curricula can indeed push students beyond the 'basics' to more in-depth problem-oriented mathematical thinking without jeopardizing their thinking in either area (Chappell, 2003, pp. 290–291).

Swafford provides analogous commentary for high school:

> Taken as a group, these studies offer overwhelming evidence that the reform curricula can have a positive impact on high school mathematics achievement. It is not that students in these curricul[a] learn traditional content better but that they develop other skills and understandings while not falling behind on traditional content. (Swafford, 2003, p.468.)

Overall, evaluations of standards-based curricula are remarkably consistent. They indicate that when they are tested on skills, students in standards-based courses perform more or less the same as (that is, with no statistically significant differences from) students who have studied traditional curricula. However, when they are tested on conceptual understanding and problem solving, the students from standards-based courses significantly outperform students who have studied traditional curricula. (Note that the curricula discussed here are precisely the ones decried in the open letter to Riley.)

A complement to the studies reported above is a large-scale comparative study conducted by the ARC Center (2003), which examined performance data on matched samples of students spread widely across Illinois, Massachusetts, and Washington State.

> The principal finding of the study is that the students in the NSF-funded reform curricula consistently outperformed the comparison students: All significant differences favored the reform students; no significant difference favored the comparison students. This result held across all tests, all grade levels, and all strands, regardless of SES and racial/ethnic identity. The data from this study show that these curricula improve student performance in all areas of elementary mathematics, including both basic skills and higher-level processes. Use of these curricula results in higher test scores (p. 5).

3.9 The impact of reform, including problem solving, as of 2008

Let us now take stock on the practical/political side. *Some* current curricula, generally known as standards-based curricula, reflect *some* of the ideas from problem-solving research. Motivated by the research of the 1970s and 1980s, those curricula devote significant attention to problem solving—although not necessarily to the use of heuristic strategies. As noted above, the field has not conducted the (quite doable) research on what it would take to learn various strategies, so such work is not reflected in standards-based curricula. What is reflected in the standards-based curricula is an understanding that problem solving is important, and that students can develop a significant amount of conceptual understanding, as well as skills, in the context of solving problems. Standards-based curricula tend to provide non-trivial opportunities for students to solve ''problems'' (as opposed to ''exercises''), to engage in mathematical reasoning, to communicate in and with mathematics, and to make mathematical connections. Sales data are difficult to obtain, but estimates are that standards-based curricula may hold about 15% of the commercial textbook market. And, what evidence there is shows that students who study from standards-based curricula tend to do at least as well, and often better, than students who study from traditional curricula.

3.10 Current national politics

Research findings and politics are two very different things. Over the course of the 1990s anti-reformers waged a very successful battle in California, ultimately gaining control over the Frameworks and textbook adoption processes. As a result, California's standards (and thus textbooks on the State's adoption list) currently emphasize teaching for mastery rather than teaching for understanding. Moreover, this is now a national trend. Not only have anti-reformers been successful nation-wide, but many of the anti-reformers who were most active during the California math wars have become advisors to the current federal administration. The federal ''No Child Left Behind'' Act has mandated testing in all 50 states, and most of that testing is skills-oriented. The US Department of Education has consistently supported a ''basics'' agenda. In early 2006 the Department, on Presidential order, formed a National Advisory Mathematics Panel (see http://www.ed.gov/about/bdscomm/list/mathpanel/index.html), whose charge it is to advise the President and Secretary of Education regarding mathematics education policy. The significant majority of the panel is traditionalist in its orientation, so it is likely that the panel report will pursue an agenda that is more in line with teaching for mastery than teaching for understanding. At present, anti-reform forces are in the ascendancy.

Recognizing this, NCTM tried to stem the tide in September, 2006 with a report entitled *Curriculum Focal Points for Kindergarten through Grade 8 Mathematics: A Quest for Coherence.* The report states clearly that it is deeply grounded in the *Standards* tradition; it says that ''*Principles and standards for school mathematics* remains the comprehensive reference on developing mathematical knowledge across the grades'' (p. 1), and every page that lists grade-level focal points (a short list of key concepts to be addressed at that grade level) contains prefatory rhetoric urging that the focal points ''be addressed in contexts that promote problem solving, reasoning, communications, making connections, and designing and analyzing representations.''

I believe that NCTM may have shot itself in the foot with the publication of *Focal Points.* The document was created for legitimate reason: in creating their state standards, many states crafted long lists of skills for students to master, without focusing on ''big ideas.'' As a result, it was hard to see coherence or direction. Focusing on a small number of key ideas could help move people toward some coherence.

However, the form of *Focal Points* may prove to be its undoing. The current (2006) California Mathematics Framework differs from the 1992 Framework in both content and form. As in the case of *Focal Points*, there is a fair amount of rhetoric about problem solving in the Framework. However, this rhetoric is not part of the definition of the content standards, which tend to be very narrowly defined and procedural in orientation. Here, for example, is the description of core skills required for the first course in algebra, called algebra I:

4 Basic skills for algebra I

The first basic skills that must be learned in algebra I are those that relate to understanding linear equations and solving systems of linear equations. In Algebra I the students are expected to solve only two linear equations in two unknowns, but this is a basic skill. The following six standards explain what is required:

> 4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.
> 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.
> 6.0 Students graph a linear equation and compute the x- and y-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequalities (e.g., they sketch the region defined by $2x + 6y < 4$).
> 7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.
> 9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

> 15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.
> (California Department of Education, 2006, p. 197)

This list of skills is very much like the list of skills in the algebra I course I took when I first studied algebra in 1959. All of the skills can be taught mechanically; none demand problem solving, reasoning, connections, or communication. Most importantly, California's textbook adoption criteria are tied to these standards, and not to the prefatory rhetoric in the Framework. Hence a textbook much like the one I studied from in 1959 would meet the standards! In fact, the California-adopted texts bear a non-trivial resemblance to those earlier texts, with ''two-page spreads'' (two facing pages, when the book is open flat on a desk) addressing one particular standard, usually via practice on procedures that are aimed directly at that standard. It is easy to imagine how such texts approach, say, standard 4.0 above: The standard is quoted verbatim, sample problems demonstrating how to simplify linear expressions are given, and then students are given practice on those skills. Page after page of the text are in this format.

The issue regarding the relationship between standards and texts is this. The discussion and exemplification in *Principles and Standards* makes it clear that the kind of skills exposition just described—a sequence of two-page spreads focusing on skills—is not compatible with the intention of *Principles and Standards.* Thus the typical California-adopted text is obviously incompatible with *Principles and Standards.* In contrast, a bare list of standards such as the one quoted above provides textbook authors with the opportunity for a sequence of narrow skills-oriented lessons.

Unfortunately, the main content of *Focal Points* (which can be downloaded in its entirety from (http://www.nctm.org/)) is presented in brief descriptions that can be treated in much the same way as the descriptions of the standards in the California Frameworks were treated. In other words (and despite the rhetoric saying that *Focal Points* are grounded in *Principles and Standards*), *Focal Points* can be used to justify the creation and adoption of texts that are very similar in style to the current skills-oriented California-adopted texts. It should come as no surprise that the traditionalists are ecstatic. Without realizing it, NCTM has played into their hands.

The initial public discussions regarding *Focal Points* have been disastrous for NCTM. For example, a *New York Times* editorial on September 18, 2006 said the following:

> One of the most infamous fads took root in the late 1980s, when many schools moved away from traditional mathematics instruction, which required drills and problem solving. The new system, sometimes derided as ''fuzzy math,'' allowed children to wander through problems in a random way without ever learning basic multiplication or division. As a result, mastery of high-level math and science was unlikely. The new math curriculum was a mile wide and an inch deep, as the saying goes, touching on dozens of topics each year.
>
> Many people trace this unfortunate development to a 1989 report by an influential group, the National Council of Teachers of Mathematics. School districts read its recommendations as a call to reject rote learning. Last week the council reversed itself, laying out new recommendations that will focus on a few basic skills at each grade level.
>
> Under the new (old) plan, students will once again move through the basics—addition, subtraction, multiplication, division and so on—building the skills that are meant to prepare them for algebra by seventh grade. (*New York Times*, September 18, 2006)

In fact, the statement that the US curriculum was ''a mile wide and an inch deep'' came from the TIMSS evaluation of the *traditional* mathematics curriculum, so the editorial is misinformed. But its tenor is clear: NCTM made a big mistake in the late 1980s, and has finally seen the light. The nation should return to the old plan.

In sum, NCTM has been outflanked by the traditionalists, and has dug itself into a very deep hole. This, combined with the skills-oriented ''high stakes testing'' that followed from the No Child Left Behind act, will make it extremely difficult, in the short run, for those who wish to support a standards-based approach to instruction. (And, I remind you, the evidence says that the approach is superior to the traditional approach! Please note that I am not saying that the standards-based curricula are as good as they should be, or that there are not significant problems with their implementation. The claim of superiority is a comparative statement based on evidence.)

5 In conclusion, a long-term view

What optimism one might have regarding the re-infusion of problem solving into the US curriculum in meaningful ways must come from taking a long-term perspective. One should recall that a decade of the ''new math''—which certainly had its flaws—was followed by an extreme reaction and a decade of the ''back to basics'' movement. Ultimately, the basics movement fell of its own weight. After a decade of skills-based instruction, students were no better at skills than they had been when the basics movement began; and (because they had not been taught them) the students were extremely poor at concepts and problem solving. When these results became evident, the need for a curricular focus on problem solving became clear.

As the philosopher George Santayana has noted, those who cannot learn from history are doomed to repeat it. It appears that much of the US has entered another ''back to basics'' movement. But one can expect this to pass, as the results of focusing almost exclusively on skills become clear (once again). In the meantime, the research community has learned a great deal about problem solving and mathematical thinking, that knowledge can be used to inform the next round of curriculum development once the climate has changed—which it inevitably will.

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