



Run Tommy Run!

RAY: This has to do with my brother.

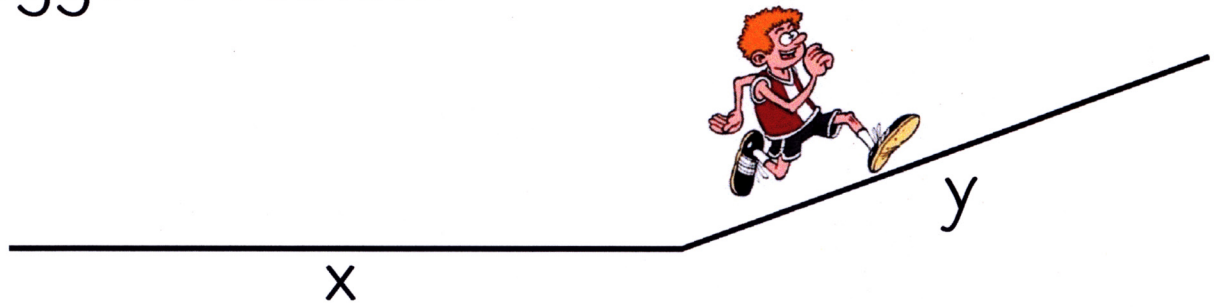
Tommy decided one day after looking in the mirror, that he needed some exercise, and he needed it now! And since his bike had two flat tires, he decided to go for a jog.

So the next morning at the crack of noon, he laces up his Keds, and begins jogging right outside his front door. First he runs on a level road, then he comes to a hill and runs to the top. When he gets to the top of the hill, he turns around and he runs back exactly the way he came.

Now, on level ground, Tommy can run at 8 miles an hour. Uphill, 6 miles an hour. And downhill, 12 miles an hour. Upon his return home, and before collapsing on the kitchen floor, he notices that he had run for exactly two hours.

So the question is, how far did he run? And it appears that there is not enough information here to solve the problem, but there is.

Jogger Problem



rate on level = 8 mph

rate up hill = 6 mph

rate down hill = 12 mph

$$d = rt, t = \frac{d}{r}$$

$$\frac{x}{8} + \frac{y}{6} + \frac{y}{12} + \frac{x}{8} = 2$$

$$\frac{2x}{8} + \frac{3y}{12} = 2$$

$$\frac{x}{4} + \frac{y}{4} = 2$$

$$x + y = 8$$

$$2(x + y) = 16$$

RAY: Well it doesn't seem like there's enough information here to solve the problem. But in fact there is because I didn't say how long the hill was. Let's assume there's no hill. The hill is a pimple.

TOM: Yeah.

RAY: OK, so you're running the whole time on level ground, and the last fraction of a second you encounter the hill. And you take one step up the hill —

TOM AND RAY: — and one step down.

RAY: So essentially you're running the entire time on level ground. Well if you ran at eight miles an hour for two hours you'll have run 16 miles.

TOM: Right.

RAY: Now let's look at the other extreme. Let's say it's all hill. So if you run up that hill at six miles an hour, and down the hill at 12 miles an hour, you will have run twice as long going up the hill as coming down the hill.

TOM: Yeah.

RAY: So if you ran for two hours it means you ran for two thirds of an hour coming down the hill, and four thirds of an hour going up the hill. Right?

TOM: Yeah.

RAY: Right, if you ran for four thirds of an hour going up the hill at six miles an hour, you ran eight miles.

TOM: Oh.

RAY: So you ran eight miles going up the hill, you obviously had to run eight miles coming down the hill so you ran a total of what?

TOM: 16.

RAY: 16 miles. So it makes no difference whether there's some hill, all hill, or no hill. Any combination of flat and hill always gives you the same result because the average of six miles an hour up the hill and 12 miles down is eight miles an hour. So you ran 16 miles.

TOM: Not bad.

RAY: How do you feel?

TOM: Ahh, I'm exhausted.