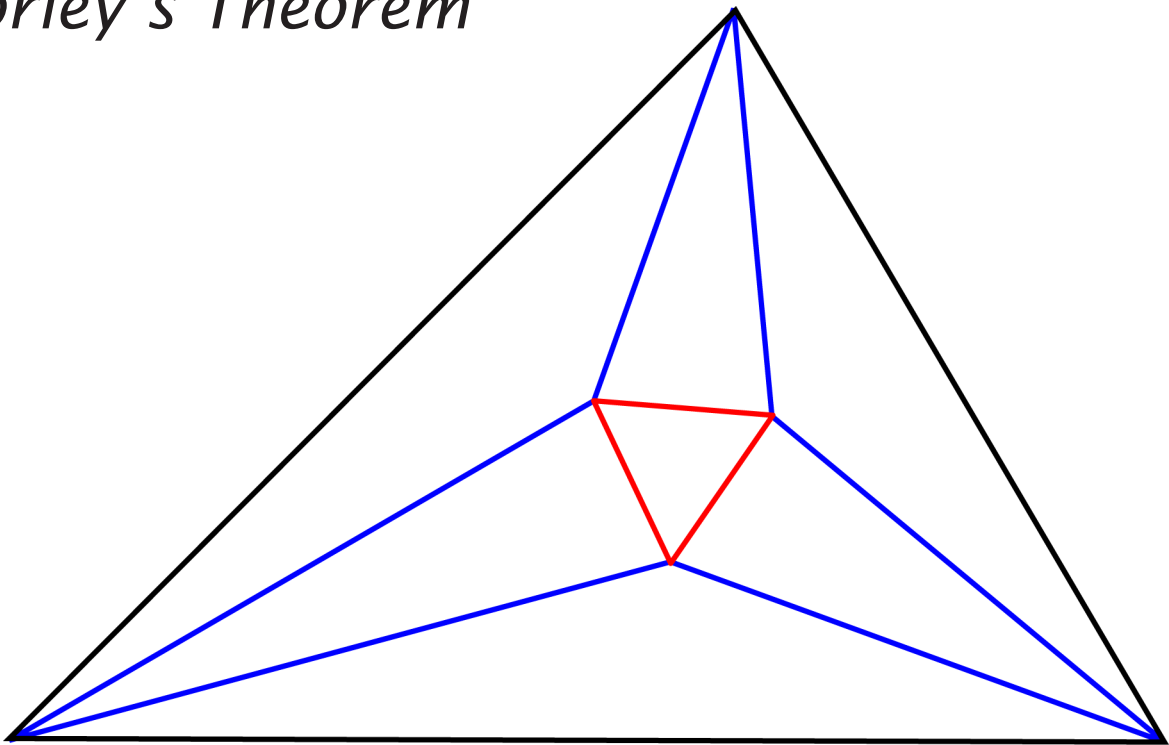




Morley's Theorem



MORLEY'S MIRACLE

A quintessential sampling of Coxeter's mathematical spirit is found in his book *Introduction to Geometry*. His opening chapter was titled "Triangles," which focused first on Euclid (perhaps a logical place to start but also a nice volley in response to the "Down-with-Euclid-Death-to-Triangles!" mantra). Therein he laid out Morley's theorem, aka Morley's Miracle, a theorem Coxeter much appreciated.¹

He also included Morley's Miracle in his book *Geometry Revisited*, coauthored with Samuel Greitzer, which featured a purple and yellow construction of the theorem on the cover (but oddly, the construction is faulty, not possessing exact trisectors).

"One of the most surprising theorems in elementary geometry"—surprising because it was so simple and went undiscovered for two thousand years—"was discovered about 1899 by Frank Morley,"² wrote Coxeter. A shy but deliberate man, Morley did not go public with his theorem and it was first published by another party, F. G. Taylor and W. L. Marr, in 1914.³ Morley was born in England, graduated from Cambridge in 1884, and later moved to the United States and became a professor of mathematics at Haverford College, in Pennsylvania; after his triangle discovery, he was appointed a professor at Johns Hopkins, in 1900.⁴

The theorem Morley discovered states: "The three points of intersection of the adjacent trisectors of the angles of any triangle form an equilateral triangle."⁵ According to Conway, "The property of equilaterality surprises everybody."⁶

Morley's son, Frank V. Morley,⁷ remembered his father's discovery: "I was a schoolboy when my father, who was almost forty years older than I was, sketched for me, free-hand, a penciled diagram of the simplest form of the above-discussed theorem in plane geometry. I tested it once with my own drawing instruments. No matter what shape of the original triangle I started with, there in its midriff was an equilateral triangle, picked out by the trisectors. It was wizard, it was weird—and it was True!"⁸

As Coxeter told the tale, Morley mentioned the theorem to his friends, who in turn spread it around the world as mathematical gossip. It was heralded as one of the most astonishing and unexpected theorems in mathematics, and a gem whose sheer beauty allows few rivals. After twenty years, Morley published his theorem in Japan. The first two proofs of the theorem included a trigonometrical proof by M. Satyanarayana and an elementary proof by M. T. Naraniengar. The theorem continued to evoke proof after proof—150 within 50 years—and still does.⁹

